

# 國立虎尾科技大學 110 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 3 頁

科目：自動控制 Automatic Control

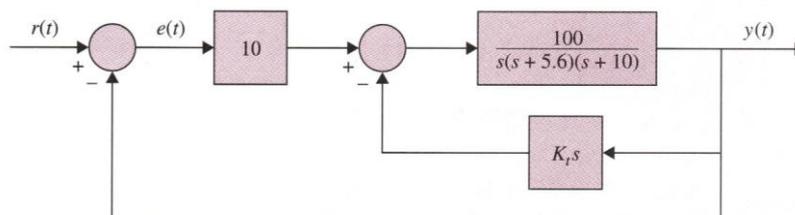
注意事項：

- (1) 本試題共有 10 題，每題 10 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止/可 使用計算機 close book and don't use calculator**

1. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

$$G(s) = \frac{1000}{(1 + 0.1s)(1 + 10s)}$$

2. The block diagram of a motor-control system with tachometer feedback is shown in the Figure. Find the range of the tachometer constant  $K_t$ , so that the system is asymptotically stable.



3. The state equations of a linear time-invariant system are represented by

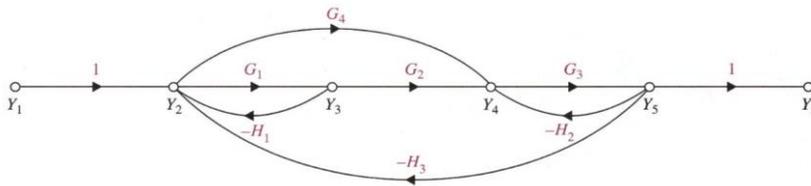
$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t)$$

Find the state transition matrix  $\phi(t)$ , the characteristic equation, and the eigenvalues of  $\mathbf{A}$  for the following cases.

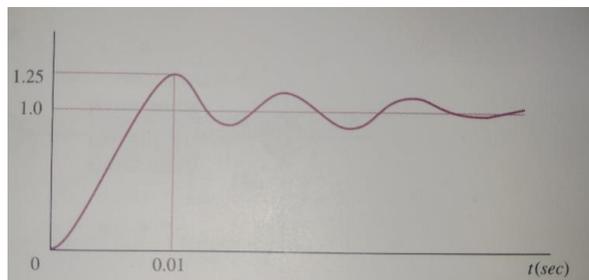
$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

4. Apply the gain formula to the SFGs shown in the Figure. find the following transfer functions:

$$\frac{Y_5}{Y_1}$$

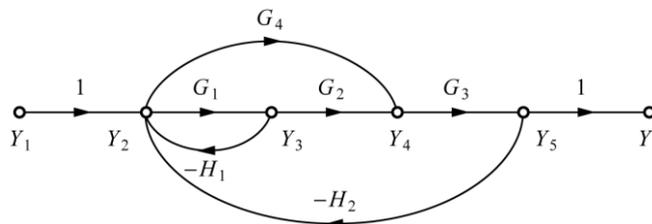


5. The unit-step response of a linear control system is shown in the Figure. Find the transfer function of a second-order prototype system to model the system.

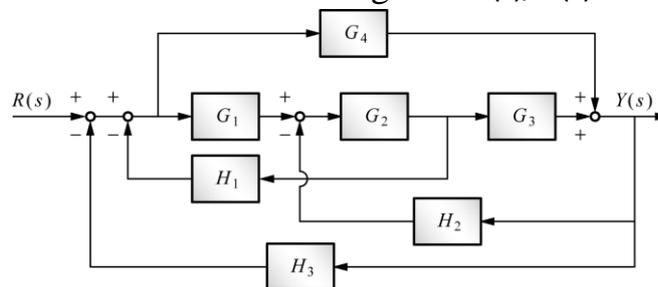


6. Find the following transfer functions in the Figure :

$$\frac{Y_5}{Y_1}$$



7. Find the following transfer functions in the Figure :  $Y(s)/R(s)$



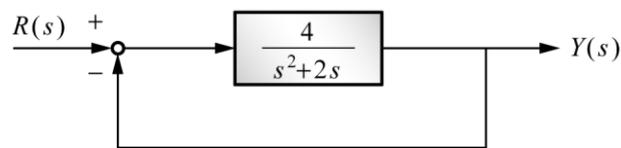
8. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^3 + 5s^2 + (K - 6)s + K = 0$$

9. The following transfer functions are given for a single-loop nonunity-feedback control system. Find the steady-state errors due to a unit-step input, a unit-ramp input, and a parabolic input,  $(t^2/2)u_s(t)$ .

$$G(s) = \frac{1}{s^2(s+12)} \quad H(s) = 5(s+2)$$

10. In the Figure. Find (a) rising time (b) peak overshoot  $M_p$  (c) damping ratio  $\zeta$  (d) natural undamped frequency  $\omega_n$  (e) damping factor  $\alpha$  (f) damping frequency  $\omega_d$



# 國立虎尾科技大學 109 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 2 頁

科目：自動控制

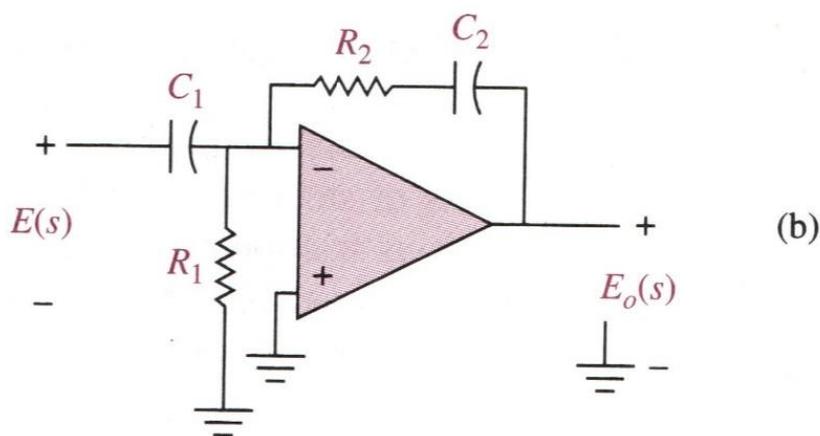
注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機與 close book

1. Solve the following differential equations by means of the Laplace transform. Assume zero initial conditions.

$$\frac{d^2 f(t)}{dt^2} + 5 \frac{df(t)}{dt} + 4f(t) = e^{-2t} u_s(t)$$

2. Find the transfer functions  $E_o(s)/E(s)$  for the circuits shown in Fig.(b)



3. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^3 + 20s^2 + 5s + 10K = 0$$

4. The state equations of a linear time-invariant system are represented by

$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t)$$

Find the state transition matrix  $\phi(t)$ , the characteristic equation, and the eigenvalues of A for the following cases.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

5. Find the steady-state errors of the following single-loop control system for a unit-step input, a unit-ramp input, and a parabolic input,  $(t^2/2)u_s(t)$ . For system that include a parameter K, find its value so that the answers are valid.

$$M(s) = \frac{s+1}{s^4+16s^3+48s^2+4s+4} \quad K_H = 1$$

# 國立虎尾科技大學 109 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 2 頁

科目：自動控制

注意事項：

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- (3) 可使用計算機與 close book

1. The following differential equations represent linear time-invariant systems. Write the dynamic equations (state equations and output equations) in vector-matrix form.

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + y(t) = 5r(t)$$

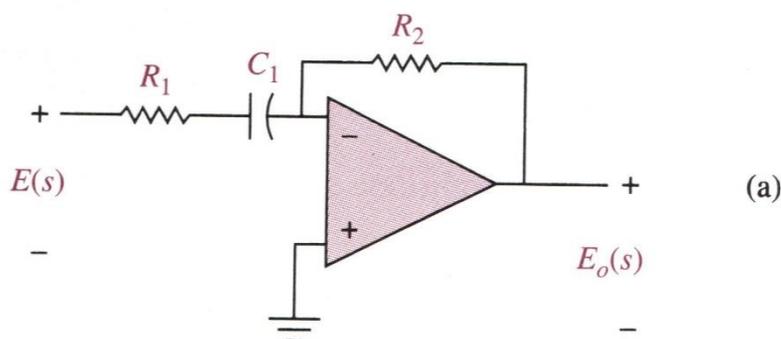
2. The state equations of a linear time-invariant system are represented by

$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t)$$

Find the state transition matrix  $\phi(t)$ , the characteristic equation, and the eigenvalues of A for the following cases.

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3. Find the transfer functions  $E_o(S)/E(s)$  for the circuits shown in Fig.(a)



4. Find the poles and zeros of the following functions (including the ones at infinity, if any). Mark the finite poles with X and the finite zeros with O in the s-plane.

$$F(s) = \frac{5(s+2)}{s(s+1)(s+3)^2}$$

5. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms if applicable.

$$f(t) = 3 + 5e^{-7t} + 2t^3, \quad (t > 0)$$

# 國立虎尾科技大學 107 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

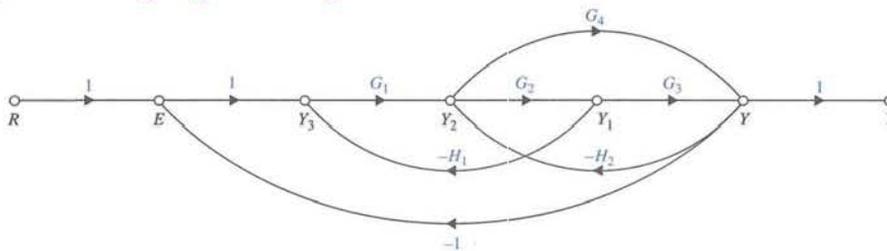
第 1 頁 共 2 頁

科目：自動控制

注意事項：

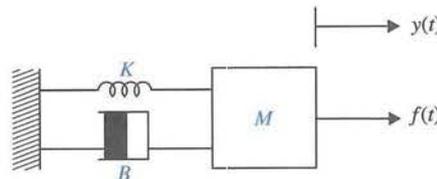
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- (3) 可使用計算機 close book

## 1. The signal-flow graph of a system is



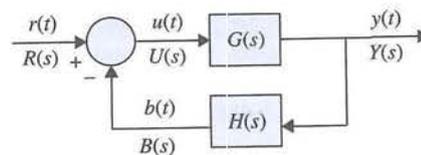
Find the transfer function of the system (  $\frac{Y(s)}{R(s)}$  ).

## 2. Consider the mass-spring-friction system



Find the transfer function between  $Y(s)$  and  $F(s)$  with zero initial conditions.

## 3. Consider the system



which has the following transfer functions:

$$G(s) = \frac{1}{s^2(s+12)} \quad H(s) = \frac{5(s+1)}{s+5}$$

Find the steady-state errors of the system for the unit-step input, the unit-ramp input and the unit-parabolic input respectively.

國立虎尾科技大學 107 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

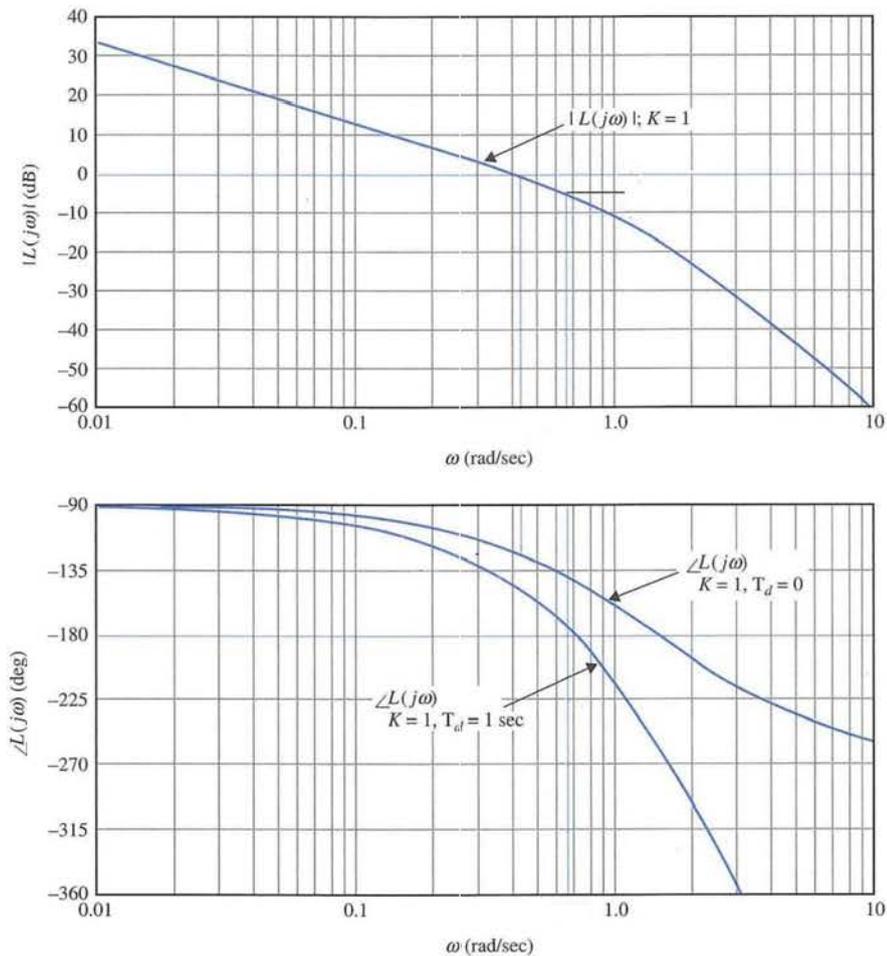
第 2 頁 共 2 頁

科目：自動控制

4. Consider that the loop transfer function of a closed-loop system is

$$L(s) = \frac{Ke^{T_d s}}{s(s+1)(s+2)}$$

The Bode plot of  $L(j\omega)$  with  $K = 1$  and  $T_d = 0$  is shown as



Find the gain margin, phase margin, gain-crossover frequency and phase-crossover frequency.

# 國立虎尾科技大學 107 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

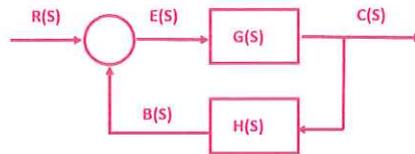
第一頁 共三頁

科目：自動控制

## 注意事項：

- (1) 本試題共有七題(第 3、4 題選一題即可)，共選六題作答，每題 16.67 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機、閉書考 hand calculator is allowed, close book test

## 第一題 (自控基本類型題):



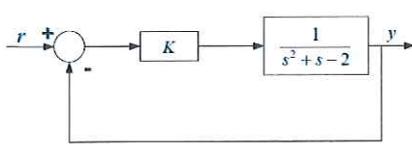
1. 求此閉迴路之轉移矩陣 M 與 G、H 關係 find relation among transfer matrix M,G and H
2. 設 G 與 H 如下，求出閉迴路轉移矩陣 M G and H given below, please find M(s)

$$G(s) = \frac{1}{s^3 + s + 1} \quad H(s) = \frac{3}{s + 3}$$

3. 利用羅斯穩定準則判斷此閉迴路系統的穩定性？ Use the Routh-criterion to judge the stability of this closed loop system

## 第二題 (數學基本功夫):

若定義  $x_1 = y, x_2 = \dot{y}$ ，系統狀態空間表示式可表示為：



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = A \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + Br$$

$$y = C \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

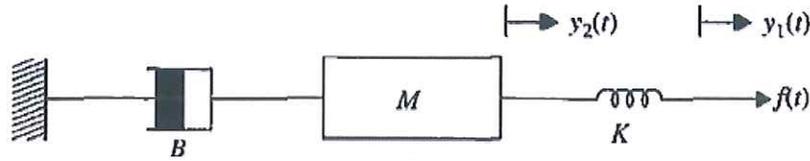
，試求：

- Please find (1) 狀態空間可控制典型式(Controllable Canonical Form)下的 A, B, C (2) A 矩陣之特徵值 A matrix eigen value (3) 由(2)之結果，試求 K 之範圍保證系統穩定 find K to ensure stability (4)使用羅斯表方法驗證(3)的結果 Use the Routh-criterion to judge (3) results

第三題 (數學模式:機): (第三題、第四題任選一題作答)

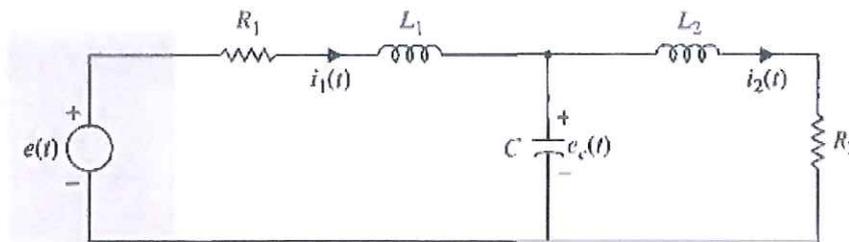
(1) Please find the state equation (first order) on this mechanical system (2) the transfer function between  $Y(s)$  and  $F(s)$

求下圖所示機械系統之(1)狀態方程式、(2)  $y(s)f(s)$ 間的轉移函數



第四題 (數學模式:電): (第三題、第四題任選一題作答)

RLC Electric network (1) find state equation of the circuit in matrix form (2) signal flow graph representation (3) The transfer functions between  $I_1(s)$  and  $E(s)$ ,  $I_2(s)$  and  $E(s)$ , and  $E_c(s)$  and  $E(s)$ , respectively 求下圖所示電子迴路

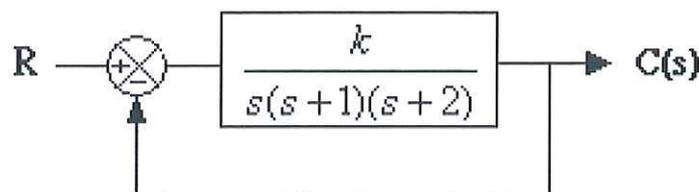


試求: (1) 求狀態方程式向量-矩陣形式 (2) 繪製狀態圖 (signal flow graph representation) (3) 轉移函數  $I_1(s)$  and  $E(s)$ ,  $I_2(s)$  and  $E(s)$ , and  $E_c(s)$  and  $E(s)$

第五題 (阻尼特性):

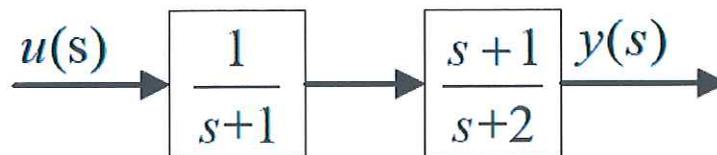
單位回授控制系統的開環路轉移函數 system transfer function  $G(s) = \frac{K}{s(s+1)(s+2)}$   $K > 0$  求

- (a) 系統具有臨界阻尼特性  $K$  的值? Please find  $K$  to make this system critically damped
- (b) 系統具阻尼比為 0.5 特性  $K$  的值? Please find  $K$  to have damping ratio 0.5



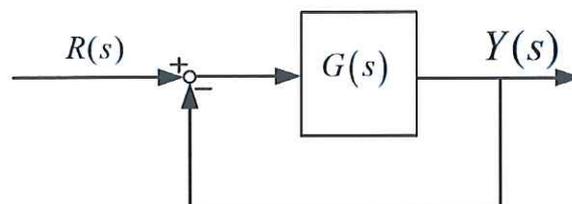
第六題

考慮下圖，以二階狀態空間描述之系統是否為可控?是否為可觀? Show below system's controllability and observability



第七題 Gain margin problem

考慮閉迴路系  $G(s) = \frac{1}{s(1+0.5s)(1+s)}$ ，求出相位邊限(Gain margin)。Find Gain margin for a given  $G(s)$ 。



# 國立虎尾科技大學 106 學年度第二學期博士班資格考試題

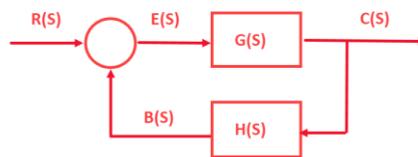
系別：動力機械工程系機械與機電工程博士班  
 科目：自動控制

第一頁 共二頁

注意事項：

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- (3) 可使用計算機 close book

第一題 (自控基本類型題):



1. 求此閉迴路之轉移矩陣  $M$  與  $G$ 、 $H$  關係 find relation among transfer matrix  $M, G$  and  $H$
2. 設  $G$  與  $H$  如下，求出轉移矩陣  $M$   $G$  and  $H$  given below, please find  $M(s)$

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s} \\ 2 & \frac{1}{s+2} \end{bmatrix} \quad H(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

第二題 (數學基本功夫):

狀態方程式如下 Consider the state equation

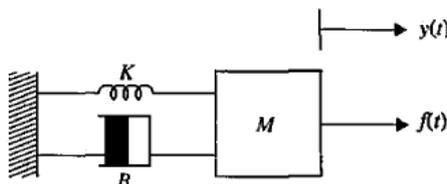
$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} dx_1(t) \\ dx_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

求解狀態轉移矩陣 (please determine state-transition matrix)  $\Phi(t)$

第三題 (數學模式:機):

- (1) Please find the state equation (first order) on this mechanical system (2) the transfer function between  $Y(s)$  and  $F(s)$

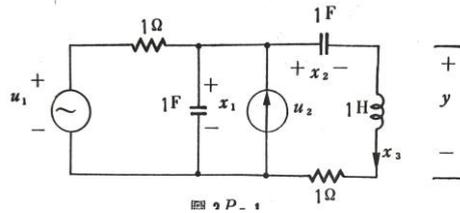
求下圖所示機械系統之(1)狀態方程式、(2)  $Y(s)F(s)$ 間的轉移函數



**第四題 (數學模式:電):**

Electric network  $u_1$  is voltage input and  $u_2$  us current source .  $Y$  is voltage output (1) find state equation of the circuit (2) state-transition matrix

求下圖所示電子迴路，有兩個輸入  $u_1$  為電壓源、 $u_2$  為電流源。電感二端電壓  $y$  為輸出



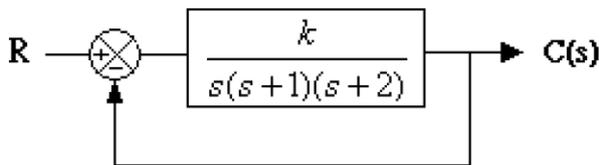
試求: (a) 迴路的動態方程式 (b) 轉移函數矩陣

**第五題 (阻尼特性):**

單位回授控制系統的開環路轉移函數 system transfer function  $G(s) = \frac{K}{s(s+1)(s+2)}$   $K > 0$  求

(a) 系統具有臨界阻尼特性  $K$  的值? Please find  $K$  to make this system critically damped

(b) 系統具阻尼比為 0.5 特性  $K$  的值? Please find  $K$  to have damping ratio 0.5



**第六題 (根軌跡圖) Root Locus Analysis**

開迴路轉移函數  $G(s) = \frac{K}{(s+4)(s+2)^2}$ ，試問點  $s = -2 + 3j$  及  $s = -1.33 + 2j$  是否位於系統閉環路的根軌跡上。需要繪圖說明。Please show if  $s = -2 + 3j$  and  $s = -1.33 + 2j$  are located in root loci. Please show on the root loci plots.

**第七題 (根軌跡圖) Root Locus Analysis**

開迴路轉移函數  $G(s) = \frac{K}{s(s+3)(s^2+2s+2)}$ ，試求下列數值。

1. 根軌跡之漸進線與時數軸之交叉點及漸進線趨向無窮處之角度 The intersection point of the progressive line of the root trajectory (root loci) with the time axis and the angle of the progressive line toward infinity
2. 求出此系統達到臨界穩定時之  $K$  值。Please find  $K$  to have system reach system critical stability
3. 匯出此系統之根軌跡圖。Root locus diagram

# 國立虎尾科技大學 106 學年度第一學期博士班資格考試題

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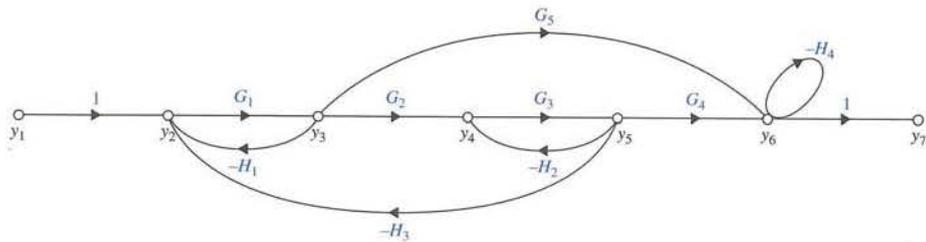
第 1 頁 共 2 頁

科目：自動控制

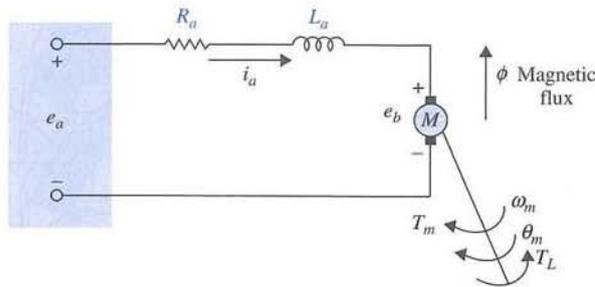
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- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機 close book

1. 試求下圖訊號流圖(Signal-Flow Graph)之輸出與輸入間的增益。



2. 試求下圖之分激式直流馬達(Separately Excited DC Motor)的狀態方程式(State Equations)與馬達位移輸出與輸入電壓間的轉移函數(Transfer Function)。



3. 試求下列狀態方程式的控制性標準型(Controllability Canonical Form)：

$$\frac{dx(t)}{dt} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

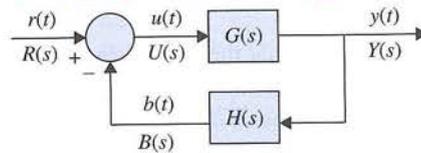
國立虎尾科技大學 106 學年度第一學期博士班資格考試題

系列：動力機械工程系機械與機電工程博士班

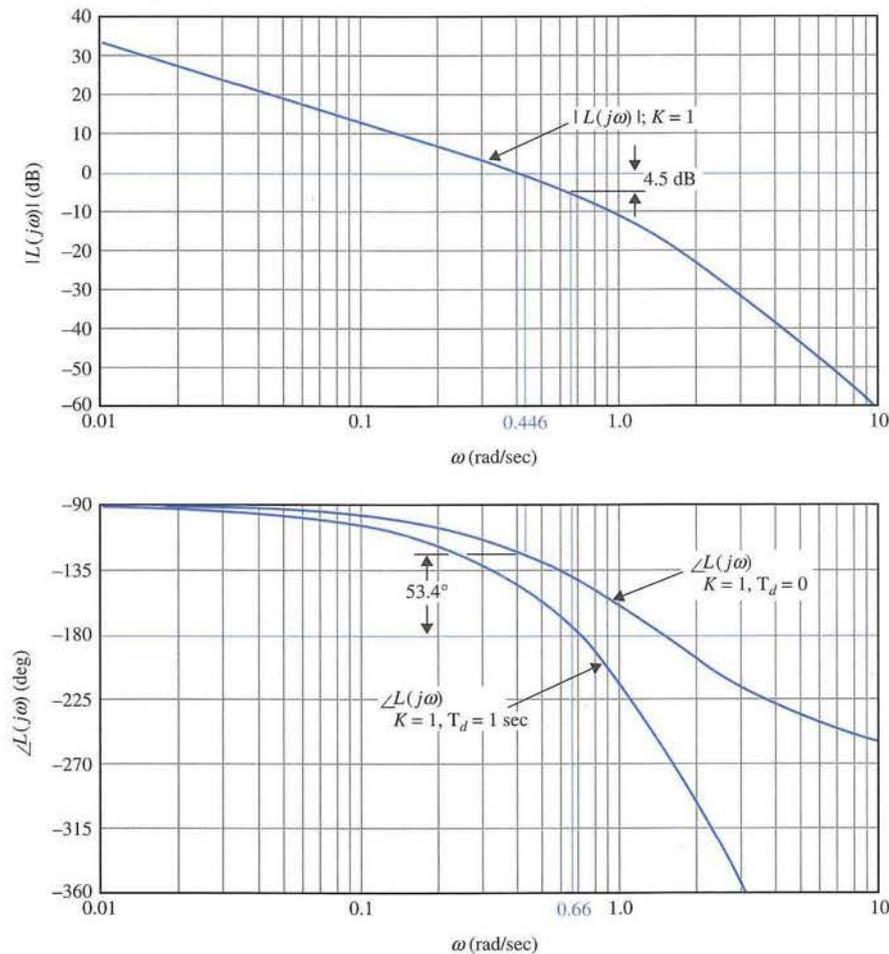
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科目：自動控制

4. 下圖回授控制系統中，轉移函數  $G(s) = \frac{1}{s^2(s+12)}$ ， $H(s) = \frac{10s}{s+5}$ 。試求單位步階輸入(Unit-Step Input)時的穩態誤差(Steady-State Error)。

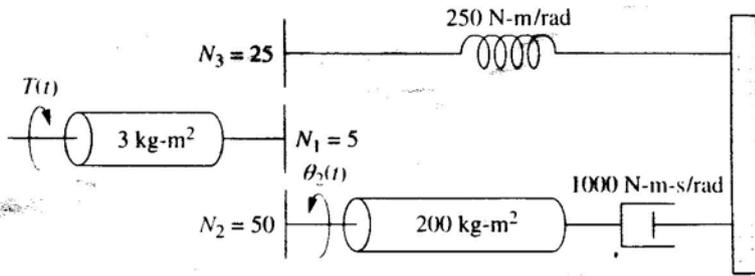


5. 一閉迴路系統(Closed-Loop System)的迴路轉移函數(Loop Transfer Function)為  $L(s) = \frac{100K(s+5)(s+40)}{s^3(s+100)(s+200)}$ ，其  $K=1$  時  $L(j\omega)$  的波德圖(Bode Plot)如下圖所示。試求系統穩定時之增益交越點頻率(Gain-Crossover Frequency)與相位邊限(Phase Margin)。



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一、如圖所示之旋轉式機械系統，試求轉移函數  $G(s) = \frac{\theta_2(s)}{T(s)}$



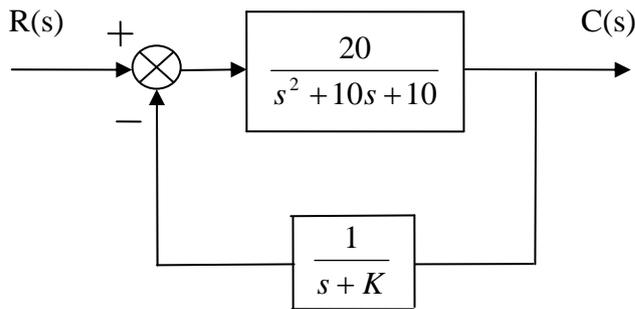
二、狀態方程式如下所示

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} X$$

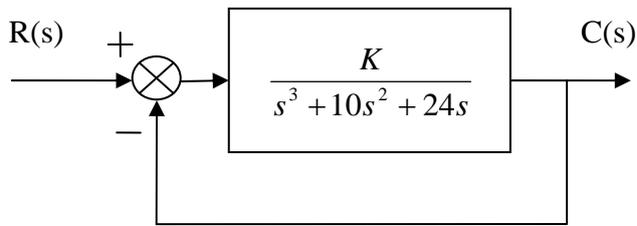
$$X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

試求狀態向量  $X(t)$

三、如圖所示控制系統，系統的輸入為單位步階，若要求穩態誤差  $e_{ss} = 0.25$ ，請求出  $K$  值。

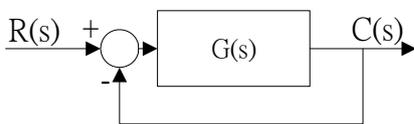


四、一單位迴授系統，其轉移函數如下所示，



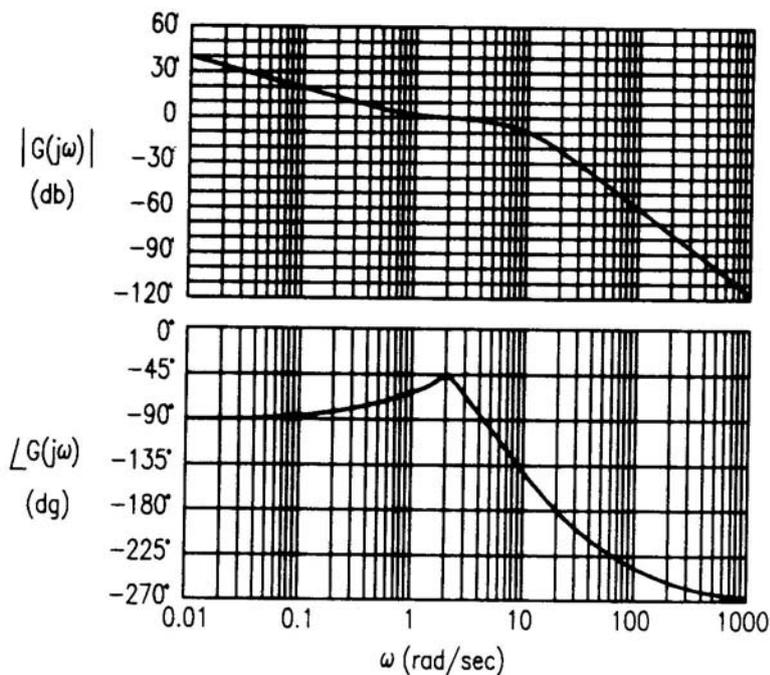
- (a) 試繪  $K > 0$  根軌跡圖
- (b) 試求根軌跡與虛軸  $j\omega$  相交的點和交點的增益  $K$  值
- (c) 使系統穩定的  $K$  值範圍。

五、單位迴授控制系統



Figure

其開迴路轉移函數  $G(s)$  的波德圖(Bode diagram)如下所示：



- (a) 試由波德圖找出增益交越頻率(gain crossover frequency)和相位交越頻率(phase crossover frequency)
- (b) 試由波德圖找出增益邊限(gain margin)和相位邊限(phase margin)
- (c) 判斷此系統是否穩定(需簡要說明理由)