

機電所博士生資格考試題 2008

工程數學(Advanced Engineering Mathematics)

每題二十分，總分一百分。

1. Diagonalize the matrix \mathbf{A} , where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Find the matrix \mathbf{P} that diagonalizes \mathbf{A} and the diagonal matrix \mathbf{D} such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$.

2. L^{-1} is the inverse Laplace transform, evaluate

$$(a) \quad L^{-1}\left\{\frac{s-1}{s^2+4}\right\}, \quad (b) \quad L^{-1}\left\{\frac{e^{-s}}{(s+1)^2}\right\}.$$

3. Solve the Bernoulli's equation

$$x^2 \frac{dy}{dx} + y^2 = xy.$$

4. Solve the Laplace's equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < a$, $0 < y < b$, subject to

$$u(x,0) = u(x,b) = 0, \quad 0 < x < a; \quad u(0,y) = 0, \quad u(a,y) = T \text{ (a constant)}, \quad 0 < y < b.$$

5. Expand $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 \leq x < \pi \end{cases}$ in a Fourier series.

國立虎尾科技大學 機電所 博士班資格考

- 1 Determine the eigenvalue and eigenvector of the problem

$Ax = \lambda Bx$, where

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

And verify that the eigenvectors are orthogonal relative to both A and B.

- 2 Solve for $x^3 y''' + x^2 y'' - 2xy' + 2y = x^3 \ln x$

- 3 $\vec{F} = \left(\frac{-y}{x^2 + y^2}\right) \vec{i} + \left(\frac{x}{x^2 + y^2}\right) \vec{j}$, find

a) $\nabla \times \vec{F}$

b) $\oint_C \vec{F} \cdot d\vec{r}$

where C is any simple closed curve, and \vec{r} is the position vector.

- 4 Please solve the following boundary value problem

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (0 < x < L, t > 0)$$

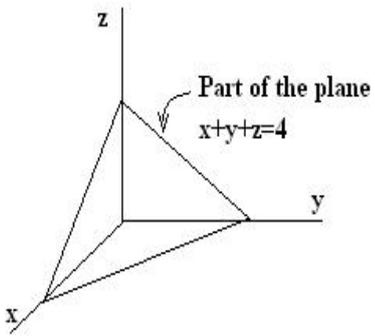
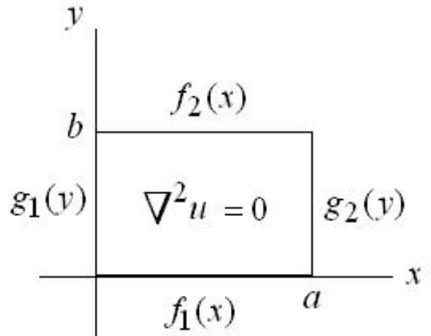
with

$$\frac{\partial T}{\partial x}(0, t) = 1$$

$$\frac{\partial T}{\partial x}(L, t) = 0$$

and

$$T(x, 0) = 0$$

<p>1. (15%)</p>	<p>A force $\vec{F}(x, y, z) = x^2 \vec{i} - zy \vec{j} + x \cos(z) \vec{k}$ move a particle along the path C given by $x = t^2$, $y = t$, $z = \pi t$ for $0 \leq t \leq 3$. The initial point is P: (0,0,0) and the terminal point of Q is Q: (9,3,3π). Suppose we want the work done in moving the particle along this path from Q to P.</p>	
<p>2. (15%)</p>	<p>Evaluate $\iint_{\Sigma} z d\sigma$ if Σ is the part of the plane $x+y+z=4$ lying above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$.</p>	
<p>3. (25%)</p>	<p>Solve the system of linear differential equations $\underline{X}' = \underline{A}\underline{X}$, where</p> $\underline{A} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad \underline{X} = [x_1, x_2, x_3, x_4], \text{ and } \underline{X}' = \frac{d\underline{X}}{dt}.$	
<p>4. (20%)</p>	<p>Solve the system of linear differential equations and initial conditions for the functions x and y:</p> $x'' - 2x' + 3y' + 2y = 4 \quad \text{and} \quad x'(0) = x(0) = y(0) = 0$ $2y' - x' + 3y = 0$	
<p>5. (25%)</p>	<p>Find the solution of partial differential problem as shown in the following Figure.</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \quad \begin{matrix} 0 \leq x \leq a \\ 0 \leq y \leq b \end{matrix}$ $u(x,0) = f_1(x) \quad u(0,y) = g_1(y)$ $u(x,b) = f_2(x) \quad u(a,b) = g_2(y)$	

學號：_____ 姓名：_____

1. 解： $2xy' + 3x^2y^2 + y = 0$

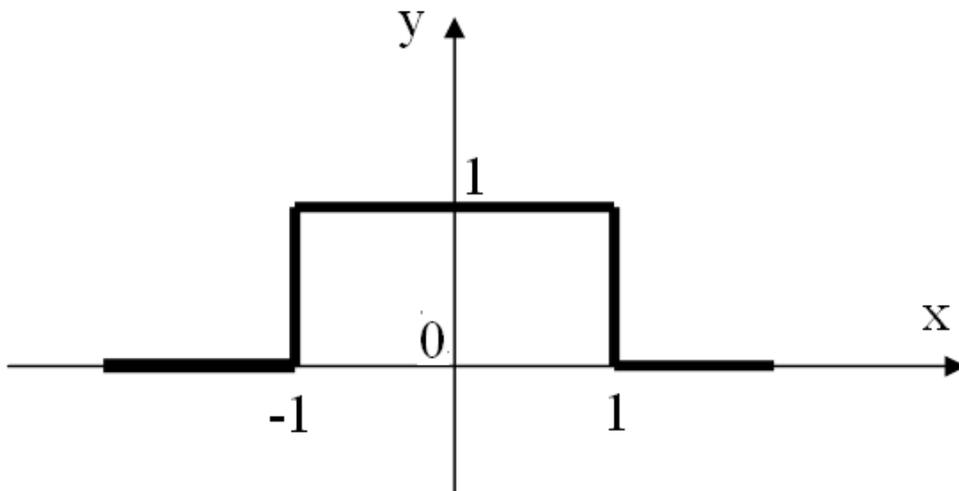
2. 解： $y'(t) = \int_0^t y(x) \cos(t-x) dx, y(0) = 1$

3. 已知 $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$, 求 e^{At}

4. 計算： $\oint_C (3y - e^{\sin x}) dx + (7x + \sqrt{y^4 + 1}) dy$,

其中 C 是圓 $x^2 + y^2 = 9$.

5. (1) 計算下圖的傅立葉積分式：



(2) 求 $\int_0^{\infty} \frac{\sin t}{t} dt$

1. 解下列微分方程式：(20%)

$$y'' - 3y' + 2y = e^{3x}$$

2. 試求下列函數之拉普拉斯轉換(Laplace Transform) (20%)

(a) $f(t) = e^{3t}t + e^{-3t}t^2$

(b) $f(t) = e^{3t}(\cos 2t + \sin 2t)$

3. 請以高斯消去法(Gauss Elimination)解下列聯立方程式(20%)

(a)
$$\begin{cases} x_1 + x_2 = 3 \\ 2x_1 - x_2 = 0 \end{cases}$$

(b)
$$\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ x_1 - 3x_2 + x_3 = 2 \\ x_1 + 3x_2 - 3x_3 = 0 \end{cases}$$

4. 解下列微分方程式：(20%)

$$y' - \frac{2}{x}y = x^2e^{-x}$$

5. 試求下列函數之拉普拉斯反轉換：(20%)

(a) $F(s) = \frac{s}{s^2 + s - 6}$

(b) $F(s) = \frac{s}{(s+2)(s-3)^2}$

1. Solve I.V.P.: $y'' - 4y' + 5y = 21e^{2x}$, $y(0) = 1$, $y'(0) = 0$. (25%)

2. Solve I.V.P. using Laplace transform: $y'' + 4y = f(t)$, $y(0) = 0$,
 $y'(0) = -1$, where $f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$. (25%)

3. Let $F(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$ and Σ be the piecewise smooth closed surface consisting of the surface Σ_1 of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 1$, together with of flat cap Σ_2 consisting of the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. Verify the Gauss's divergence theorem, i.e. $\iint_{\Sigma} F \cdot N d\sigma = \iiint_V \nabla \cdot F dV$. (25%)

4. Find the eigenvalues of matrix A and the corresponding eigenvectors. (25%)

$$A = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

國立虎尾科技大學 機械與機電工程研究所

100 學年度第 2 學期 博士班資格考 【工程數學】

1. 試求出下列微分方程式之解：

10% (a) $y'' - 2y' - 3y = 2\sin^2 x$

10% (b) $y' = 2 + \left(\frac{y}{x}\right)^2$

15% (c) $y'' + 5y' + 6y = f(t); y(0) = 0, y'(0) = -4$

$$f(t) = \begin{cases} t^2 & , \text{if } 0 \leq t < \pi \\ \cos t & , \text{if } t \geq \pi \end{cases}$$

2. 已知 $f(x) = \frac{1}{2}x^2$ for $-\pi \leq x \leq \pi$

14% (a) 試求出其 Fourier series

6% (b) 並以(a)之答案求出 $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} = \frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \dots = ?$

3.

5% (a) 已知 $f(x) = H(x)e^{-ax}$ ($a > 0$), $H(x)$ 為 Heaviside step function, 試求 $f(x)$ 之 Fourier transform, $F[f(x)] = ?$

10% (b) 試求 $F[e^{-a|x|}] = ?$ 即 $e^{-a|x|}$ 之 Fourier transform。

10% (c) 試求 $F^{-1}[e^{-|w|} \cos w] = ?$ 即 $e^{-|w|} \cos w$ 之 Fourier inverse transform。

4. 已知 $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

5% (a) 試求 A^{-1} (A 之反矩陣)。

5% (b) 試求 A 之 eigenvalue 及 eigenvector。

10% (c) 試求 $A^{30} = ?$

國立虎尾科技大學 102 學年度第一學期博士班資格考試題

所別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 題，任選 題作答，每題 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止/可 使用計算機

solving the following differential equation。

$$(1) 2xyy' + (x-1)y^2 = x^2e^x$$

$$(2) x^2y'' - 2xy' + 2y = 16x^3\cos x$$

(3) using Laplace transform to solve the following differential equation。

$$y'' + 3y' + 2y = 4t^2 \text{ if } 0 < t < 1 \text{ and } 8 \text{ if } t > 1;$$

$$y(0) = 0, y'(0) = 0$$

(4) find the following function $f(x)$ of the Fourier integral.

$$f(x) = e^{-|x|} (-\infty < x < \infty)$$

(5)

(a). find the following function $f(x)$ of line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ using the region R boundary

C to calculate by counter clockwise

$F = [6y^2, 2x-2y^4]$, R is the square of the four points $\pm(2, 2)$ and

$\pm(2, -2)$ 。

(b). calculate the above problem by using the green threom to find integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ 。

國立虎尾科技大學 102 學年度第二學期博士班資格考試題

所別：動力機械系機械與機電工程博士班

第 1 頁 共 2 頁

科目：工程數學

注意事項：

- (1) 本試題共有 4 題，依題配分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機，Close Book。

25% (1) Please solve the following differential equation

$$11\% \text{ (a)} 2yy' + y^2 \sin x = \sin x, \quad y(0) = \sqrt{2}$$

$$14\% \text{ (b)} x^2 y'' - 2y' + y = x \ln|x|$$

$$y(1) = 1, \quad y'(1) = -4$$

15% (2) Please use laplace transform to solve the following equation

$$y'' + 9y = f(t) \quad f(t) = \begin{cases} 0 & 0 \leq t \leq \pi \\ \cos t & t \geq \pi \end{cases}$$

40% (3)

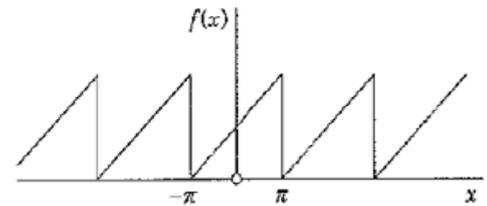
15% (a) find the Fourier series of the Function (Fig.1)

$$f(x) = f(x + 2\pi)$$

5% (b) using the above the answer, find the following series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \dots (-1)^n \frac{1}{2n+1} \dots = ?$$

20% (c) Solve the following differential equation



(Fig.1)

$$y'' - 4x = f(x) \quad , \quad f(x) \text{ is the function of Fig.1}$$

$$20\% \text{ (4)} \quad [A] = \begin{bmatrix} -6 & 11 & 3 \\ 4 & 1 & 3 \\ -4 & 10 & 8 \end{bmatrix}$$

8% **(a)** find the eigenvalue and eigenvector of $[A]$, one of eigenvalue of $[A]$ is

$$\lambda_1 = 2$$

12% **(b)** find $[A]^{20} = ?$

國立虎尾科技大學 103 學年度第一學期博士班資格考試題

所別：動力機械系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
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- (3) 禁止使用計算機，close book

1. Solve the following problems.

a. $y'' - 6y' + 9y = 5e^{3x}$; b. $x^2 y'' - 4xy' + 6y = x^4 e^x$; $y(2) = 2, y'(2) = 7$

2. Solve the following problems by the Laplace Transform

a. $y'' + 4y = f(t); f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}; y(0) = y'(0) = 0;$

b. $y'' + 2y' + 2y = \delta(t - 3); y(0) = y'(0) = 0;$ where $\delta(t)$ is the Dirac Delta function.

3. Find the Fourier series of the following functions.

a. $f(x) = x$ for $-\pi \leq x \leq \pi;$

b. $f(x) = \begin{cases} 0 & \text{for } -3 \leq x \leq 0 \\ x & \text{for } 0 \leq x \leq 3 \end{cases}$

4. Solve the following Nonhomogeneous systems of linear equations

$$x_1 - x_3 + 2x_4 + x_5 + 6x_6 = -3$$

a. $x_2 + x_3 + 3x_4 + 2x_5 + 4x_6 = 1$

$$x_1 - 4x_2 + 3x_3 + x_4 + 2x_6 = 0$$

b.
$$\begin{pmatrix} 2 & 1 & -11 \\ -5 & 1 & 9 \\ 1 & 1 & 14 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ 12 \\ -5 \end{pmatrix}$$

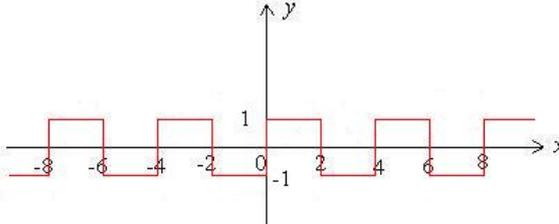
國立虎尾科技大學 103 學年度第二學期博士班資格考試題

所別：動力機械系機械與機電工程博士班本試題

科目：工程數學

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- (3) 禁止計算機，本試題為 close book 考試

1.	<p>Solve the following Ordinary Differential equations:</p> <p>(a). $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$,</p> <p>(b) $y'' - 2y' + y = x^2 e^x$, and</p> <p>(c) $x^2 y'' - 5xy' + 8y = 2\ln(x) + x^3$</p>
2	<p>Expand $f(x) = \begin{cases} 1, & 0 \leq x \leq 2 \\ -1, & -2 \leq x \leq 0 \end{cases}$ and $f(x+4) = f(x)$ into the Fourier series. Then, find the values of the following series: (a) $\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)}$ and (b) $\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}$.</p> 
3	<p>Solve $y'' + y - 4 \int_0^t y(\tau) \sin(t - \tau) d\tau = e^{-2t}$, where $y(0) = 1$ and $y'(0) = 0$.</p>
4	<p>Solve the system of Linear differential equations</p> $\begin{cases} x_1'' + 10x_1 - 4x_2 = 0 \\ 4x_1 + x_2'' + 4x_2 = 0 \end{cases}, \quad \text{where the initial conditions are given as } \begin{cases} x_1(0) = 0 & x_1'(0) = 1 \\ x_2(0) = 0 & x_2'(0) = -1 \end{cases}$
5	<p>Prove the matrix $\underline{A} = \begin{bmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 1 & 0 & 0 \\ 0 & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$ is orthogonal.</p>

國立虎尾科技大學 104 學年度第 1 學期博士班資格考試題

系別：動力機械系機械與機電工程博士班

科目：工程數學

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- (3) 禁止使用計算機且 close book

1. Solve the initial value problem (IVP): $y'' - 3y' + 2y = x + 2$; $y(0) = 0$, $y'(0) = 0$.

2. Find the initial value problem (IVP): $x^2y'' - 5xy' + 8y = 2\ln(x)$; $y(1) = 0$, $y'(1) = 0$.

3. Use "**Laplace transform**" to solve the initial values problem (IVP): $y'' + 4y = f(t)$; $y(0) = 0$, $y'(0) = 0$, in which $f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}$.

4. Find the eigenvalues and the corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}.$$

5. A function $f(x) = x^2$ is defined on $[-\pi, \pi]$. Find its Fourier series on the defined interval.

國立虎尾科技大學 104 學年度第 2 學期博士班資格考試題

系別：動力機械系機械與機電工程博士班

科目：工程數學

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- (3) 禁止使用計算機且 close book

1. Solve the initial value problem (IVP) : $y'' + 4y' - 2y = 2x^2 - 3x + 6$; $y(0) = -9$, $y'(0) = 0$.

2. Find the initial value problem (IVP): $x^2 y'' - 4xy' + 6y = \ln(x^2)$; $y(1) = \frac{5}{18}$, $y'(1) = 0$.

3. Use "Laplace transform" to solve the initial value problem (IVP): $y'' - 2y' + 5y = 1 + t$; $y(0) = 0$, $y'(0) = 4$.

4. Solve the equations of system by any matrix technique.

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 7 \\ x_1 - x_2 + 3x_3 = 3 \\ 5x_1 + 4x_2 - 2x_3 = 1 \end{cases}$$

5. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \leq x \leq \pi \end{cases}$ in a Fourier series.

國立虎尾科技大學 105 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

注意事項：

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- (3) 禁止使用計算機 且 close-book exam

1. Solve the initial value problem (IVP) : $y'' + y' - 6y = 2x$; $y(0) = 1$, $y'(0) = 0$.

2. Solve the initial value problem (IVP) : $x^2y'' + 10xy' + 8y = x^2$; $y(1) = 0$, $y'(1) = 0$.

3. Use “**Laplace transform**” to solve the initial values problem (IVP): $y'' - 3y' + 2y = e^{-4t}$;
 $y(0) = 1$, $y'(0) = 5$.

4. Find the eigenvalues and the corresponding eigenvectors for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.

5. Find the half-range cosine and sine expansions of the given function $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1/2 \\ 0 & \text{for } 1/2 \leq x < 1 \end{cases}$.

國立虎尾科技大學 106 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

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- (3) 禁止使用計算機 且 close book

1. (a) Solve the initial value problem (IVP): $y' + \frac{1}{x}y = -\frac{2}{x}$; $y(1) = 1$.
(b) Solve the initial value problem (IVP): $4xdx + 9ydy = 0$; $y(2) = 0$.

2. Solve the initial value problem (IVP) : $y'' - 2y' + y = 2\sin(x)$; $y(0) = 0$, $y'(0) = 0$.

3. Use “**Laplace transform**” to solve the initial values problem (IVP): $y'' - 5y' + 6y = H(t - 1)$; $y(0) = 0$, $y'(0) = 1$. (Note: $H(t - 1)$ is a unit step function)

4. Find the rank of the given matrix $A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{bmatrix}$.

5. Expand $f(x) = x^2$, $0 < x < L$,
 - (a) in a cosine series
 - (b) in a sine series.

國立虎尾科技大學 106 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 8 題，每題 12.5 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機 close book

1. Solve(求解微分方程式)

$$y' = \frac{y(y-2)}{x(y-1)}$$

2. Solve(求解微分方程式)

$$\sin y dx + (x \cos y - 2y)dy = 0$$

3. Solve(求解微分方程式)

$$y'' + 4y' + 7y = 0$$

4. Solve(求解微分方程式)

$$x^3 y''' - 3x^2 y'' + 7xy' - 8y = 0$$

5. Known(已知)

$$L\{\sin at\} = \frac{a}{s^2+a^2}, \quad (s > 0)$$

Evaluate(求) $L\{t \sin at\}$

6. By the elementary row operations(使用基本列運算), to evaluate the inverse

matrix of $\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (求取反矩陣)。

7. Solve the eigenvalues and eigenspaces of $\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (試求矩陣的特徵值與特徵空間)。

8. Solve(求解方程組)

$$\begin{aligned} x' &= x + 4y - 4t^2 - 3, & x(0) &= 2 \\ y' &= x + y - t^2 + 2t - 3, & y(0) &= 3 \end{aligned}$$

國立虎尾科技大學 107 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機 close book

1. (a) Solve the differential equation(求解微分方程式)

$$dx + (3x - e^{-2y})dy = 0$$

(b) Solve the differential equation(求解微分方程式)

$$y'''' - y'' = 0$$

2. To evaluate (求解) $L^{-1}\left(\ln\frac{s-a}{s-b}\right)$, where a and b are real numbers.

3. Determine all eigenvalues and eigenspaces (求矩陣的特徵值與特徵空間) of

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

4. Solve the system (求解方程組)

$$\begin{aligned} x' &= x + 4y - 4t^2 - 3, & x(0) &= 2 \\ y' &= x + y - t^2 + 2t - 3, & y(0) &= 3 \end{aligned}$$

5. Expand the square wave function defined below in Fourier series (試求下列方波函數之 Fourier 級數展開).

$$f(x) = \begin{cases} 0, & x \in (-\pi, 0) \\ 4, & x \in (0, \pi) \end{cases}$$

$$f(x + 2\pi) = f(x)$$

國立虎尾科技大學 107 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，任選○題作答，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止 使用計算機**

1. Given

$$y'' + 2y' + 2y = r(t) \quad \begin{cases} r(t) = 10 \sin 2t & \text{if } 0 < t < \pi \\ r(t) = 0 & \text{if } t > \pi \end{cases} \quad \begin{cases} y(0) = 1 \\ y'(0) = -5 \end{cases}$$

- (1) Solve the initial value problem above using Laplace Transfer method.
- (2) What mechanical system can apply above differential equation?

2. At point P in O_{xi} coordinate axes the stress tensor is

$$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Find values of principal stresses (i.e., eigen values) and the directions associated with them (i.e., principal axes)

3. (1) Please find Fourier Series of $f(t)$ below

$$f(t) = \begin{cases} -1 & \text{if } -2 < t \leq 0 \\ 1 & \text{if } 0 < t \leq 2 \end{cases} \quad T=4$$

(2) Please find Fourier Transfer of $f(x)$, "a" is any constant.

$$f(x) = e^{-a|x|}$$

4. Please solve differential equation below

(1) $(2x - 4y + 5)y' + x - 2y + 4 = 0$

(2) $xy' + 2y = 4e^{x^2}$

5. Find reverse Matrix of A (i.e., A^{-1})

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

- (1) Using Gauss-Jordan Elimination
- (2) using Determinants (i.e., Det A)

國立虎尾科技大學 108 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 大題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 且 close book

1. Solve I.V.P. $y'' + 7y' + 12y = 12x + 19; y(0) = 1, y'(0) = 0$.

2. Use Laplace transform to solve I.V.P. $y'' - 3y' - 4y = H(t - 1); y(0) = 2, y'(0) = 4$. (Note: $H(t-1)$ is a unit step unction)

3. Diagonalize the matrix $A = \begin{bmatrix} -2 & 7 & 3 \\ 6 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

4. Let $f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ \pi - x, & 0 < x \leq \pi \end{cases}$ and $f(x) = f(x + 2\pi)$. Expand $f(x)$ in Fourier series.

5. Let a vector field $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$.
 - (a) Show \vec{F} is a conservative field.
 - (b) Find the potential function of \vec{F} .
 - (c) Calculate the work done by \vec{F} from $(0, 1, -1)$ to $(\pi/2, -1, 2)$.

國立虎尾科技大學 108 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 大題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 且 close book

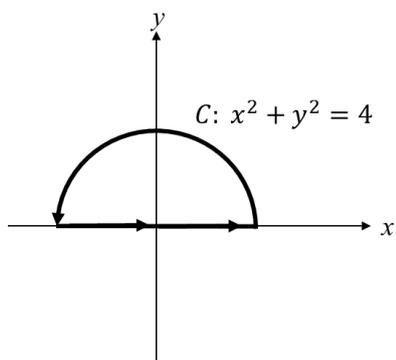
1. Solve I.V.P. $y'' + 4y = 3 \sin(2x); y(0) = 1, y'(0) = 0$.

2. Use Laplace transform to solve I.V.P. $y'' - 2y' + 5y = 1 + t; y(0) = 0, y'(0) = 4$.

3. Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$.

4. Find the half-range cosine and sine expansions of the given function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$.

5. Evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ on the given closed curve C.



國立虎尾科技大學 109 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止 使用計算機 close book**

1. Solve I.V.P. $y'' + 2y' = 2x + 5 - e^{-2x}$, $y(0) = 2, y'(0) = 0$.

2. Use Laplace transform to solve I.V.P. $y'' - 4y' = 6e^{3t} - 3e^{-t}$, $y(0) = 1, y'(0) = -1$.

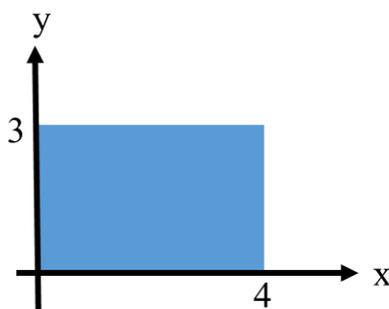
3. Find the eigenvalues and the corresponding eigenvectors of matrix A.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

4. Find the Fourier series of function $f(x)$ on the given interval.

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$$

5. Find the center of mass of the lamina that has the given shape and the area density $\rho(x, y) = xy$.



國立虎尾科技大學 109 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 2 頁

科目：工程數學 (Engineering Mathematics)

注意事項：

- (1) 本試題共有五題，每題註明配分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止使用計算機 close book and don't use calculator**

1. Find solutions of a system of simultaneous linear equations below (30%)

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ -x_1 + x_2 + x_3 = 0 \\ 3x_1 - 2x_2 - x_3 = 2 \end{cases}$$

(1) Please write in standard matrix format $AX=B$ (2) Using Gauss-Jordan Elimination (3) Using reverse Matrix of A (i.e., A^{-1})

2. (a) Please find solution of initial value problem below (20%)

$$\begin{cases} yy'' - (y')^2 = 0 \\ y(0) = \frac{1}{2}, y'(0) = 1 \end{cases}$$

(b) Please find solution of differential equation below

$$y'' = \frac{1 + (y')^2}{2y}$$

3. (a) Using Laplace transform solve a system of differential equations, (20%)

$$\begin{cases} x' = 2x - 3y \\ y' = -2x + y \end{cases}, x(0) = 8, y(0) = 3$$

(b) Solve the problem using Laplace transform

$$y'' + 4y' + 4y = 1 + \delta(t - 1), \quad y(0) = 0, y'(0) = 2$$

4. A block of unit mass (with $m=1$) under external force F acting on x -axial, .

$$F = -A\omega^2 \sin(\omega t)$$



As motion starts, this block Q positions on zero location with velocity $A\omega$. Please find Q movement (as function of time). (10%)

5. (a) What is "Fourier Series" used for? (20%)

(b) Please find Fourier Series

$$f(t) = 1 + t; \quad -1 < t < 1$$

(c) Please find the sum of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots \dots$$

國立虎尾科技大學 110 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學 Engineering Mathematics

注意事項：

- (1) 本試題共有五題，每題二十分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 **close book and don't use calculator**

1. Solve I.V.P. (a) $y' - 2xy = 2x, y(0) = 1$. (Hint: Linear)

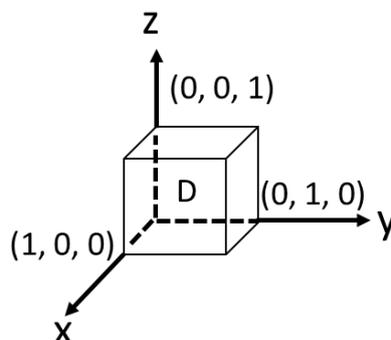
(b) $(x^2 - y)dx + (y - x)dy = 0, y(0) = 1$. (Hint: Exact)

2. Solve I.V.P. $y'' + 4y = 2x, y(0) = 1, y'(0) = 1$.

3. Use the Laplace transfer to solve $y'' - 2y' - 3y = f(t), y(0)=1, y'(0)=0$, in which $f(t) =$

$$\begin{cases} 0, t < 4 \\ 3, t \geq 4 \end{cases}$$

4. Verify the Gauss divergence theorem of the vector field, $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$, for the region D bounded by the unit cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.



5. Find the half-range (a) Fourier-cosine (b) Fourier-sine expansions of the function, $f(x)$, on the given interval. $f(x) = x^2 + x, 0 < x < 1$

國立虎尾科技大學 110 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有五題，每題二十分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 close book and don't use calculator

1. (a) Solve $y' + y = e^{-x}, y(0) = 1$. (*Hint: Linear*)
(b) Solve $(x^2 + y)dx + (x - y)dy = 0, y(0) = 2$. (*Hint Exact*)

2. Solve Euler's differential equation: $x^2y'' + 3xy' + 13y = 4 + 3x$ for $x > 0$.
(*Hint: Let $x = e^t$, i.e. $t = \ln(x)$*)

3. Use Laplace transform to solve: $y'' - 4y' = 6e^{3t} - 3e^{-t}, y(0) = 1, y'(0) = -1$.

4. Find the eigenvalues and the corresponding eigenvectors of matrix A.
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where the vector field \vec{F} is $\vec{F} = xy^2\hat{i} + (x^2y + y^3)\hat{j}$ and the curve C is $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}, 0 \leq t \leq \frac{3\pi}{2}$.
(b) Find the length S of the curve C: $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + \sqrt{3}t\hat{k}, 0 \leq t \leq 2\pi$.
(*Hint: $S = \int_C dS = \int_C \left| \frac{d\vec{r}}{dt} \right| dt$*)

國立虎尾科技大學 111 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號。
- (3) 可使用計算機，close book, may use calculator,

1. Solve the given differential equation:

$$x^2 y'' + 3xy' + 17y = 4 + 3x$$

2. Find solution of a simultaneous equation, (1) using Gauss-Elimination method; (2) using inverse of matrix method.

$$\begin{cases} 0.3x_1 + 0.52x_2 + x_3 = -0.01 \\ 0.5x_1 + x_2 + 1.9x_3 = 0.67 \\ 0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44 \end{cases}$$

3. Using Laplace transform to solve the IVP problem:

$$y'' - 4y' = 6e^{3t} - 2e^{-t}, y(0)=1, y'(0) = 4 .$$

4. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} \pi^2, & -\pi < x < 0 \\ \pi^2 - x^2, & 0 \leq x < \pi \end{cases}$$

5. Find the half-range cosine and sine expansions:

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$

國立虎尾科技大學 111 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 4 題，每題 25 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號。
- (3) 可使用計算機， close book, may use calculator ,

1. Solve the given differential equation:

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 3 + \ln x^3$$

2. Find solution of a simultaneous equation, (1) using Gauss-Elimination method;
(2) using inverse of matrix method.

$$\begin{cases} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{cases}$$

3. Using Laplace transform to solve the initial value problem(IVP):

$$y'' - 3y' + 2y = e^{-4t}, y(0)=1, y'(0) = 5 .$$

4. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$$